TOPIC 7

Algebraic Concepts & Techniques

Checklist
Have you understood the following?
1. Algebraic manipulation
2. Equations
3. Inequalities
4. Indices and logarithms

Algebraic Manipulation

- The word substitution means replacing a letter by a number.
- After replacing the letter, the expression is then simplified as far as possible using the order of operations.
- You may be required to substitute values into formulae such as in Science and in the calculation of area and volume.

Guided Question 7.1

Given that $x=4$, and $z=-2$ find the value of the following when $y=-1$:

a) $x(4-y)-z$

b) $\dfrac{3(z-y)-2(x+z)}{x+y}$

Solutions

a) Replace x by 4, y by -1 and z by -2 then have

$$\begin{aligned} x(4-y)-z &= 4(4-(-1))-(-2) \\ &= 4(4+1)+2 \\ &= 4(5)+2 \\ &= 20+2 \\ &= 22. \end{aligned}$$

b) To simplify fractions, one must always simplify the numerator on its own and the denominator on its own then combine them:

$$\begin{aligned} \frac{3(z-y)-2(x+z)}{x+y} &= \frac{3(-2-(-1))-2(4-2)}{4-1} \\ &= \frac{3(-2+1)-2(2)}{3} \\ &= \frac{3(-1)-4}{3} \\ &= \frac{-3-4}{3} \\ &= -\frac{7}{3} \\ &= -2\frac{1}{3}. \end{aligned}$$

It is possible to add and subtract algebraic terms as we do in arithmetic with known numbers.

Simplification of algebraic expressions can be made easier by grouping positive terms together first and then negative terms together.

Like terms are those algebraic terms with the same letter. Unlike terms are those with different letters. 13*k*, *k* and –6*k* are **like terms** because they are in the same letter *k*.

On the other hand, 7, 7*y* and 7*k* are **unlike terms** because they have different letters even though they involve the same number 7.

The terms are grouped together as for positive and negative terms. The sum or difference of unlike terms cannot be simplified and is left as it is.

Guided Question 7.2

Simplify

a) $K-\frac{3}{8}k+2k-\frac{1}{8}k$

b) $p+q-\frac{1}{6}p-\frac{1}{4}q$

c) $\frac{4}{5}x-\frac{1}{4}a-\frac{1}{5}x-a$

Solutions

a) $K-\frac{3}{8}k+2k-\frac{1}{8}k$ can be simplified by grouping positive terms together and simplifying their coefficients.

When positive terms are grouped together, we have $K+2k-\frac{3}{8}k-\frac{1}{8}k$

The coefficients are $1+2-\frac{3}{8}-\frac{1}{8}$

$\therefore K-\frac{3}{8}k+2k-\frac{1}{8}k=\frac{8+16+3+1}{8}k$

$=\frac{20}{8}k=2\frac{1}{2}k$

b) $p+q-\frac{1}{6}p-\frac{1}{4}q$ becomes $p-\frac{1}{6}p+q-\frac{1}{4}q$ after grouping like terms. Simplifying the coefficients, we have $\left(1-\frac{1}{6}\right)p+\left(1-\frac{1}{4}\right)q$

$\left(\frac{6-1}{6}\right)p+\left(\frac{4-1}{4}\right)q$

$\left(\frac{5}{6}\right)p+\left(\frac{3}{4}\right)q$

c) $\frac{4}{5}x-\frac{1}{4}a-\frac{1}{5}x-a$ re-arranges to become $\frac{4}{5}x-\frac{1}{5}x-\frac{1}{4}a-a$

The coefficients of *x* easily simplify to $\frac{4}{5}-\frac{1}{5}=\frac{3}{5}$

$\therefore$ the term in *x* is $\frac{3}{5}x$

The expression $-a-\frac{1}{4}a$ means that the total of $1+\frac{1}{4}=1\frac{1}{4}$ are being subtracted.

The final answer becomes $\frac{3}{5}x-1\frac{1}{4}a$

Multiplication and Division

The product of a and b is shown simply by writing $a \times b = ab$.

The order in which the numbers are multiplied does not matter.

In algebra, $2 \times a \times 4 = 2 \times 4 \times a = 4 \times 2 \times a = 8a$.

Remember that the coefficient 8 is written first.

Division in algebra works as it does in arithmetic. Common factors cancel out.

Guided Question 7.3

Simplify

a) $h \times 2h \times h$ b) $4n \times 5$ c) $5xy^2 \times 3x^2$

d) $22y \div 11$ e) $\frac{3}{5}$ of \$35 m f) 20% of 450 MW

g) $48x^2y \div 16xy$ h) $4x^3 \div x^2$ i) $7x$% of 300 mb

Solutions

a) $h \times 2h \times h = 2 \times h \times h \times h = 2h^3$

b) $4n \times 5 = 4 \times 5 \times n = 20 \times n = 20n$

c) $5xy^2 \times 3x^2 = 5 \times 3 \times x \times x^2 \times y^2 = 15 \times x \times x\, xy^2 = 15x^3y^2$

d) $22y \div 11 = \frac{22y}{11} = \frac{2 \times 11y}{11} = 2y$

e) $\frac{3}{5}$ of $\$35 = \frac{3}{5} \times \$35 = 3 \times \$7 = \$21$ m

f) 20% of 450 MW $= \frac{20}{100} \times 450$ MW $= \frac{1}{5} \times 450$ MW = 90 MW

g) Common factors cancel out from the numerator and denominator.

$$48x^2y \div 16xy = \frac{48x^2y}{16xy} = \frac{3 \times 16 \times x \times x \times y}{16xy} = 3x$$

h) Common factors cancel out from the numerator and denominator.

$$4x^3 \div x^2 = \frac{4 \times x \times x \times x}{x \times x} = 4x$$

i) $7x\%$ of 300 mb $= \frac{7x}{100} \times 300$ mb $= \frac{7x \times 3 \times 100}{100 \text{ mb}} = 7x \times 3$ mb $= 21$ mb = 21 mb

Quick Self-Test Questions 7.1

1. Find the sum of d dollars and $10d$ cents giving your answer in cents.

2. Simplify

a) $x - 10x + 18x - 3x$ b) $\frac{64a^2b^2}{16ab^2}$ c) $\frac{5b-(b+3b)\div 2}{3b}$

d) $(a+5a) \times 2 + 5 \times 3a - (a+4a)$ e) $at \times at$ f) $-63mtn \div 9m$

g) $x + \left(1\frac{1}{2}x + \frac{1}{2}x\right) \times 2$

3. Simplify

a) $\frac{1}{6}$ of $(-24m)$ b) $(-4w)^2$ c) $-\frac{2}{3}$ of $(-15mn)$

4. Simplify

a) $6y - (4.2x - 2.2x) \div 2$ b) $\frac{4mk}{3} \times \left(-\frac{9k}{4}\right)$

5. Simplify

a) 30% of $150xy$ b) $2x$% of $200y$

6. A student bought 10 writing exercise books at x cents each and a Mathematics textbook for \$$5y$. How much did the student pay? Express your answer in dollars.

7. A farmer asked a mechanic to repair his water pump. The mechanic charged \$$0.5x$ for labour. Spare parts cost \$$x$ and gasket sealer cost \$$0.1x$.
What was the total cost of repairs?

Simplify the following:

8. a) $2y - 5y + 21y - 12y$ b) $10 + 3a - 9 + 7a - 8a$

9. a) $4b - \frac{b + 5b}{3}$ b) $1 + 2e + \frac{e + 3e}{4}$

10. a) $-\frac{2}{3}$ of $15xz$ b) $-\frac{1}{4}$ of $(-16d)$

Equations

Subject of formula

Guided Question 7.4

Make h the subject of the formula in $g - \frac{h}{x} = 0$

Solution

$g - \frac{h}{x} = 0$

$g(x) - \frac{h}{x}(x) = 0 \times x$ (multiply both sides by x)

$gx - h = 0$

$gx - h + h = 0 + h$ (add h to both sides)

$gx = h$

Guided Question 7.5

Make x the subject of the formula in $v = \frac{\pi x^3}{6}$

Solution

$6(v) = \frac{\pi x^3}{6} \times 6$ (multiply both sides by 6)

$6v = \pi x^3$

$\frac{6v}{\pi} = \frac{x^3}{\pi}$ (divide both sides by π)

$\frac{6v}{\pi} = x^3$

$\sqrt[3]{\frac{6v}{\pi}} = \sqrt[3]{x^3}$ (cubing both sides)

$\sqrt[3]{\frac{6v}{\pi}} = x$

Guided Question 7.6

Make y the subject of the formula in $s = \frac{xy^2}{k} - 3xg$

Solution

$s = \frac{xy^2}{k} - 3xg$

$s(k) = (k)\frac{xy^2}{k} - 3xg(k)$ (multiply both sides by k)

$sk = xy^2 - 3xgk$

$sk + 3xgk = xy^2 - 3xgk + 3xgk$ (add $3xgk$ on both sides)

$sk + 3xgk = xy^2$

$\frac{sk + 3xgk}{x} = \frac{xy^2}{x}$ (divide both sides by x)

$\frac{sk + 3xgk}{x} = y^2$

$\sqrt{\frac{sk + 3xgk}{x}} = \sqrt{y^2}$ (Squaring both sides)

$\sqrt{\frac{sk + 3xgk}{x}} = y$

Guided Question 7.7

It is given that $s = ut - \frac{1}{2}gt^2$. Make g the subject of the formula.

Solution

$$s = ut - \frac{1}{2}gt^2$$

$$s(2) = (2)ut - (2)\frac{1}{2}gt^2 \qquad \text{(multiply both sides by 2)}$$

$$2s = 2ut - gt^2$$

$$2s + gt^2 = 2ut - gt^2 + gt^2 \qquad \text{(add } gt^2 \text{ both sides)}$$

$$2s + gt^2 = 2ut$$

$$2s - 2s + gt^2 = 2ut - 2s \qquad \text{(subtract 2s from both sides)}$$

$$gt^2 = 2ut - 2s$$

$$\frac{gt^2}{t^2} = \frac{2ut - 2s}{t^2} \qquad \text{(divide by } t^2 \text{ on both sides)}$$

$$g = \frac{2ut - 2s}{t^2}$$

Quick Self-Test Questions 7.2

Make b the subject of the formula in the following algebra:

1. $n = \frac{b-5}{3b-2}$

2. $A = \frac{1}{2}bh$

3. $x = \frac{1}{2}\sqrt{a^2 - b^2}$

Simple Algebraic Fractions

Lowest terms

Reducing algebraic fractions entails full factorisation of both the numerator and the denominator.

Guided Question 7.8

Express as a single fraction in its lowest terms

$$\frac{3}{x^2 - x} - \frac{5}{x^2 - 1}$$

Solution

$$\frac{3}{x^2-x}-\frac{5}{x^2-1}$$

$$=\frac{3}{x(x-1)}-\frac{5}{(x-1)(x+1)}$$

$$=\frac{3(x+1)-5x}{x((x-1)(x+1))}$$

$$=\frac{3x+3-5x}{x((x-1)(x+1))}$$

$$=\frac{3-2x}{x(x^2-1)}$$

$$=\frac{3-2x}{(x^3-x)}$$

Guided Question 7.9

Reduce $\frac{mxw}{xwv}$ to its lowest terms.

Solution

$$\frac{mxw}{xwv}=\frac{mxw}{xwv}=\frac{m}{v}$$

Factors, Multiples, HCF, LCM

A combination of an unknown and a numeral, e.g., $6y$, $19y$, $-3y$ are called terms in y.The three have y in common hence are **like terms**.

The product of 2 and 6 is 12. 2 and 6 are **factors** of 12. If we have 2 and $(x+y)$ their product is $2(x+y)$ or $2x+2y$. 2 and $(x+y)$ are factors of $2x + 2y$.

HCF – Highest Common Factor

The HCF is the highest number which is a factor of 2 or more numbers (otherwise it wouldn't be common).

Guided Question 7.10

Find the HCF of 15 and 9.

Solution

15 = 3 x 5

9 = 3 x 3

OR

Listing factors of 15 and 9 as follows:

Factors of 15: 15, 5, 3, 1

Factors of 9: 9, 3,3,1

Common factors are 1 and 3 and the HCF is 3

HCF = 3

Guided Question 7.11

Find the HCF of 200 and 530.

Solution

200 = 2 x 2 x 2 x 5 x 5

530 = 2 x 5 x 53

HCF = 2 X 5

= 10

Guided Question 7.12

Find the HCF of $24x^3y$ and $36x^3yz^3$

Solution

$24x^3y = 2 \times 2 \times 2 \times 3 \times x \times x \times x \times y$

$36x^3yz^3 = 2 \times 2 \times 3 \times 3 \times x \times x \times x \times y \times z \times z \times z$

$\text{HCF} = 2 \times 2 \times 3 \times x \times x \times x \times y$

$= 12\ x^3y$

LCM – Lowest Common Multiple

This is the lowest number which is a multiple of two or more numbers.

Guided Question 7.13

Find the LCM OF 9 and 3.

Solution

$9 = 3 \times 3$

$3 = 3 \times 1$

$\text{LCM} = 3 \times 3 \times 1$

$= 9$

OR

Listing the first 5 multiples of 9 and 3, we get

Multiples of 9: 9, 18, 27, 36, 45, 54, 63…

Multiples of 3: 3, 6, 9, 12, 15, 18, 21…

9 and 18 are the common multiples of 9 and 3 but 9 is the lowest common multiple (LCM).

Guided Question 7.14

Find the LCM of $2x^2$, $3y$ and $16p$.

Solution

$2x^2 = 2 \times x \times x$

$3y = 3 \times y$

$16p = 2 \times 2 \times 2 \times 2 \times p$

$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 3 \times x \times x \times y \times p$

$= 48x^2yp$

Expansion

- Expanding brackets involves removing the brackets from an expression by multiplying out the brackets. This is achieved by multiplying every term inside the bracket by the term outside the bracket.
- When multiplying out double brackets, every term in the first pair of brackets must be multiplied by each term in the second.
- When expanding brackets, be very careful when dealing with negative numbers.

Guided Question 7.15

a) Expand the expression $2(x+y)$.

b) Expand and simplify $(3x+2y)(2x-y)$.

c) Expand $(n-3)(m+8)$.

Solution

a) $2(x+y) = 2x + 2y$

b) $(3x+2y)(2x-y) = 3x(2x-y) + 2y(2x-y)$

$= 6x^2 - 3xy + 4xy - 2y^2$

$= 6x^2 + xy - 2y^2$

c) $(n-3)(m+8) = n(m+8) - 3(m+8)$

$= mn + 8n - 3m - 24$

Factorisation

Factorising is the reverse process of expanding brackets. To factorise an expression fully, means to put it in brackets by taking out the highest common factors.

The simplest way of factorising is:

- Find the highest common factor of each of the terms in the expression.
- Write the highest common factor (HCF) in front of any brackets.
- Fill in each term in the brackets by multiplying out.

Guided Question 7.16

Factorise $a^2b + ab^2$

Solution

Step 1: HCF of a^2b and ab^2 is ab

Step 2: Dividing the expression by HCF

$\sqrt[ab]{a^2b+ab^2}$

$a+b$

$\therefore a^2b + ab^2 = ab(a+b)$

Indices

Application of the laws of Indices

We define $a^1 = a$

Theorem 1: The product rule

If m and n are natural numbers, then its true to say that $a^m \times a^n = a^{m+n}$

Guided Question 7.17

Simplify

a) $a^6 \times a^3$

b) $3^5 \times 3^2$

Solution

a) $a^6 \times a^3 = a.a.a.a.a.a.a.a.a = a^9$

This can be written in short as:

$a^6 \times a^3 = a^{6+3} = a^9$

b) $3^5 \times 3^2 = 3^{5+2} = 3^7 = 2\ 187$

Theorem 2

Assume that m and n are natural numbers, $a \neq 0$ and $m > n$;

$$a^m \div a^n = \frac{a^m}{a^n}$$

Guided Question 7.18

Simplify

a) $a^3 \div a^2$

b) $10^5 \div 10^3$

c) $a^{10} \div a^{12}$

Solution

a) $a^3 \div a^2 = \frac{a^3}{a^2} = \frac{a.a.a}{a.a} = a$

b) In short, $\frac{a^3}{a^2} = a^3 \div a^2 = a^1 = a$

Hence, we have the rule: $a^m \div a^n = a^{m-n}$

$10^5 \div 10^3 = 10^5 - 3 = 10^2$

c) $a^{10} \div a^{12} = a^{10-12} = a^{-2}$

Theorem 3

If $a \neq 0$, then $a^0 = 1$

Assume that m and n are natural numbers, $a \neq 0$ and $m = n$

$$\frac{a^m}{a^n} = \frac{a^m}{a^m} = 1$$

Guided Question 7.19

a) $\frac{a^3}{a^3} = 1$

Solution

a) $\frac{a^3}{a^3} = a^{3-3} = a^0$

But $\frac{a^3}{a^3} = \frac{a.a.a}{a.a.a} = 1$

$\therefore a^0 = 1$

Theorem 4 – (The Power Rule)

$(a^m)^n = a^{mn}$

$(a^m)^n \ = a^m.a^m.a^m \ldots.\ a^m \ = a.a.a.a\ldots.a = a^{mn}$

Guided Question 7.20

Simplify

a) $(a^3)^2$

b) $\left(3^{\frac{1}{2}}\right)^2$

Solution

a) $(a^3)^2 = a^{3.2} = a^6$

b) $\left(3^{\frac{1}{2}}\right)^2 = 3^{\frac{1}{2}.2} = 3^1 = 3$

Theorem 5

$a^{\frac{1}{n}} = \sqrt[n]{a}$

Guided Question 7.21

Evaluate

a) $32^{\frac{1}{5}}$

b) $81^{\frac{1}{2}}$

Solution

a) $32^{\frac{1}{5}} = \sqrt[5]{32} = 2$

b) $81^{\frac{1}{2}} = \sqrt[2]{81} = 9$

Theorem 6

$a^{-n} = \frac{1}{a^n}$

Proof

$a^3 \div a^5 = a^{3-5} = a^{-2}$

But $\frac{a^3}{a^5} = \frac{a.a.a}{a.a.a.a.a} = \frac{1}{a^2}$

Hence, $a^{-2} = \frac{1}{a^2}$

Guided Question 7.22

Simplify

a) $(6a)^{-2}$

b) $\left(\frac{1}{4}\right)^{-2}$

Solution

a) $(6a)^{-2} = \frac{1}{(6a)^2} = \frac{1}{36a^2}$

b) $\left(\frac{1}{4}\right)^{-2} = \frac{1}{\left(\frac{1}{4}\right)^2} = \frac{1}{\frac{1}{16}} = \frac{16}{1} = 16$

Or $\left(\frac{1}{4}\right)^{-2} = \left(\frac{4}{1}\right)^2 = \frac{16}{1} = 16$

Theorem 7

$$\frac{A^m}{n} = \left(n\sqrt{a}\right)^m$$

Guided Question 7.23

Evaluate

a) $\left(\left(\frac{256}{16}\right)^{-2}\right)^{\frac{1}{4}}$

b) $\left(\left(\frac{27}{48}\right)^{-3}\right)^{\frac{1}{2}}$

Solution

a) $\left(\left(\frac{256}{16}\right)^{-2}\right)^{\frac{1}{4}} = \sqrt[4]{\left(\frac{16}{256}\right)^2} = \left(\frac{2}{4}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

b) $\left(\left(\frac{27}{48}\right)^{-3}\right)^{\frac{1}{2}} = \frac{\left(\frac{48}{27}\right)^3}{2} = \frac{\left(\frac{16}{9}\right)^3}{2} = \left(\frac{4}{3}\right)^3 = \frac{64}{27} = 2\frac{10}{27}$

Equations

An equation is a mathematical statement, which has an equal sign (=) between the algebraic expression.

Linear equations

Linear equations are the equations of degree 1. It is the equation for the straight line. The solutions of linear equations will generate values, which when substituted for the unknown values, make the equation true. In the case of one variable, there is only one solution. For example, the equation $x + 2 = 0$ has only one solution as $x = -2$. But in the case of the two-variable linear equation, the solutions are calculated as the Cartesian coordinates of a point of the Euclidean plane.

Guided Question 7.24

Determine the domain of x and the solution set for the equation $6x + 4 = 10$.

Solution

$6x + 4 = 10$

$6x + 4+(-4) = 10+(-4)$ (add –4 to both sides)

$6x = 6$

$x = 1$ (divide by 6 both sides)

The domain of x is a set of all real numbers and the solution set for the equation $6x + 4 = 10$ is $\{1\}$.

Guided Question 7.25

Solve the equation $4x + 8 = 24$

Solution

$4x+8 = 24$

$4x+8+(-8) = 24+(-8)$ (add –8 to both sides)

$4x = 16$ (combine terms)

$\frac{4x}{4} = \frac{16}{4}$ (divide both sides by 4)

$x = 4$ (simplify)

Guided Question 7.26

Solve $2(3x - 2) = 4x + 5$ for x.

Solution

The use of the distributive rule is important for the removal of the brackets.

$2(3x - 2) = 4x + 5$

$6x - 4 = 4x + 5$ (remove brackets)

$6x - 4 + 4 = 4x + 5 + 4$ (add 4 to both sides)

$6x = 4x + 9$ (combine terms)

$6x + (-4x) = 4x + (-4x) + 9$ (add $-4x$ to both sides)

$2x = 9$ (combine terms)

$x = \frac{9}{2}$ (divide both sides by 2)

$x = 4\frac{1}{2}$ (simplify)

Guided Question 7.27

Solve the equation

$$\frac{-x^2+3}{x^2-1}+\frac{7x}{x+1}=\frac{6x}{x-1}$$

Solution

$$\frac{-x^2+3}{x^2-1}+\frac{7x}{x+1}=\frac{6x}{x-1}$$

$$\frac{-x^2+3}{(x-1)(x+1)}+\frac{7x}{x+1}=\frac{6x}{x-1} \qquad \text{factor } x^2-1$$

$$\frac{(-x^2+3)(x-1)(x+1)}{(x-1)(x+1)}+\frac{7x(x-1)(x+1)}{x+1}=\frac{6x(x-1)(x+1)}{x-1} \qquad \text{(multiply both sides by } (x-1)(x+1))$$

$$(-x^2+3)+7x(x-1)=6x(x+1) \qquad \text{(simplify)}$$

$$-x^2+3+7x^2-7x = 6x^2+6x \qquad \text{(remove brackets)}$$

$$6x^2+3+7x = 6x^2+6x \qquad \text{(combine terms)}$$

$$3-7x = 6x \qquad \text{(add } -6x^2 \text{ to both sides)}$$

$$-13x=-3 \qquad \text{(add } -3 \text{ to both sides)}$$

$$\frac{-13x}{-13}=\frac{-3}{-13} \qquad \text{(divide by } -13 \text{ to both sides)}$$

$$x=\frac{3}{13} \qquad \text{(simplify)}$$

Simultaneous equations

- The simultaneous equation is an equation that involves two or more quantities that are related using two or more equations.
- It includes a set of few independent equations.
- The simultaneous linear equations can be solved using various methods.
- The substitution and elimination methods.

Substitution

- The elimination method is one of the techniques to solve the system of linear equations.
- In this method, either add or subtract the equations to get the equation in one variable.
- If the coefficients of one of the variables are the same, and the sign of the coefficients are opposite, we can add the equation to eliminate the variable.
- Similarly, if the coefficients of one of the variables are the same, and the sign of the coefficients are the same, we can subtract the equation to get the equation in one variable.

Guided Question 7.28

Solve the following simultaneous equations using the substitution method:

$4x - 2y = 5$

$x + y = -\frac{1}{4}$

Solution

$4x - 2y = 5$ (1)

$x + y = -\frac{1}{4}$ (2)

Make y the subject in (1);

$\frac{4x-5}{2} = y$ (3)

Substitute $\frac{4x-5}{2}$ for y in (2);

$x + \frac{4x-5}{2} = -\frac{1}{4}$

$2x + 4x - 5 = -\frac{1}{2}$ (multiply by 2 throughout)

$6x = -\frac{1}{2} + \frac{5}{1}$ (combine terms)

$6x = \frac{9}{2}$ (simplify)

$12x = 9$ (multiply by 2 to both sides)

$x = \frac{9}{12} = \frac{3}{4}$ (simplify)

Substitute $\frac{3}{4}$ for x in (3)

$\frac{4\left(\frac{3}{4}\right) - 5}{2} = y$

$\frac{3-5}{2} = y$

$-\frac{2}{2} = y$ or $y = -1$

The solution set is $x = \frac{3}{4}$, $y = -1$

Guided Question 7.29

Solve the following simultaneous equation:

$3x + 2y = 0$

$2x + y = -1$

Solution

$3x + 2y = 0$ (1)

$2x + y = -1$ (2)

Multiply (1) by 1 and (2) by 2

$\times 2 \quad 2x + y = -1$ (2)

$\times 1 \quad 3x + 2y = 0$ (3)

$4x + 2y = -2$ (4)

Subtract (4) from (3) to eliminate y;

$-x+0=2$

$x=-2$

substitute –2 for x in (1);

$3(-2) + 2y = 0$
$-6 + 2y = 0$
$2y = 6$
$y = 3$

The solution is $x = -2$ and $y = 3$.

Quadratic Equations

A quadratic equation in x is any equation that can be written in the standard form:

$ax^2+bx+c=0$ a, b, $c \in$ natural numbers $a \neq 0$.

There are four methods of solving quadratic equations namely,

i) factorisation
ii) using the formula
iii) completing the square
iv) graphical

Solving by factorisation is based on the result that if $a \times b=0$, then either $a=0$ or $b=0$ (Zero product property).

Guided Question 7.30

Solve the equation $x^2 + 5x - 24 = 0$

Solution

Step 1: $(x-3)(x+8)=0$ (factor the left side of the equation)

Step 2: $x-3=0$ or $x+8=0$ (zero-product property)

$x = 3$ or $x = -8$

Step 3: Check the roots by substituting them in the original equation.

$3^2 + 5(3) - 24 = 0$ $\quad$ $(-8)^2 + 5(-8) - 24 = 0$

$9 + 15 - 24 = 0$ $\quad$ $64 - 40 - 24 = 0$

$\therefore$ the solution set is (–8; 3).

Remember

1) Zero product property states that for all real numbers a and b, $ab=0$, if and only if, $a=0$ or $b=0$.
2) The quadratic formula: The solution of $ax^2+bx+c=0$ is given by the equation:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Guided Question 7.31

Solve $8x^2 - 2x - 15 = 0$

Solution

Apply the quadratic formula : $a = 8$; $b = -2$ and $c = -15$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(8)(-15)}}{2(8)} = \frac{2 \pm \sqrt{484}}{16} = \frac{2 \pm 22}{16} = -1\frac{1}{4} \text{ or } 1\frac{1}{2}$$

Inequalities

Unlike equations, inequalities have more than one solution. The set of all numbers that satisfy a given inequality is called a solution set.

Here are some of the useful results/axioms about inequalities. If $a, b, c \in R$, then

a) $a < b$ and $b < c$ implies $a < c$ and we write $a < b < c$

b) $a < b, c > 0$ implies $ac > ab$

c) $a < b$, $c < 0$ implies $ac > ab$

Guided Question 7.32

Solve the inequality $3(3x - 10) < 5$ and represent it on a number line.

Solution

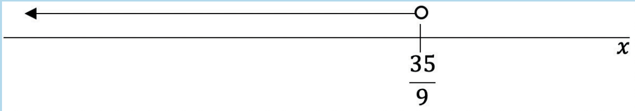
Solve the inequality as if it were an equation

$3(3x - 10) < 5$

$9x - 30 < 5$ (remove brackets)

$9x < 35$ (add 30 to both sides)

$x < \frac{35}{9}$ (divide both sides by 9)



Guided Question 7.33

Solve the linear equation $-3(2x + 2) \leq 12$ and illustrate it on the number line.

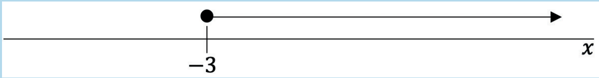
Solution

$-3(2x + 2) \leq 12$

$-6x - 6 \leq 12$ (remove brackets)

$-6x \leq 18$ (add 6 to both sides)

$x \geq -3$ (divide both sides by -6)



Guided Question 7.34

Solve the inequality $-2 \le 3x + 4 < 7$ illustrating it on a number line.

Solution

$-2 \le 3x + 4$ $3x + 4 < 7$ (split the two equations and work separately)

$-6 \le 3x$ $3x < 3$

$-\frac{6}{3} \le \frac{3x}{3}$ $\frac{3x}{3} < \frac{3}{3}$

$-2 \le x$ $x < 1$

$\therefore -2 \le x < 1$

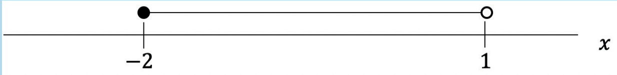
OR

Work with the given inequality without splitting.

$-2 \le 3x + 4 < 3$

$-6 \le 3x < 3$ (add –4 to all the 3 terms)

$-2 \le x < 1$ which means x is any number between -2 and 1



Guided Question 7.35

Using graph paper, show the regions defined by the following inequalities (use solid lines and broken lines where appropriate and leave each required region unshaded):

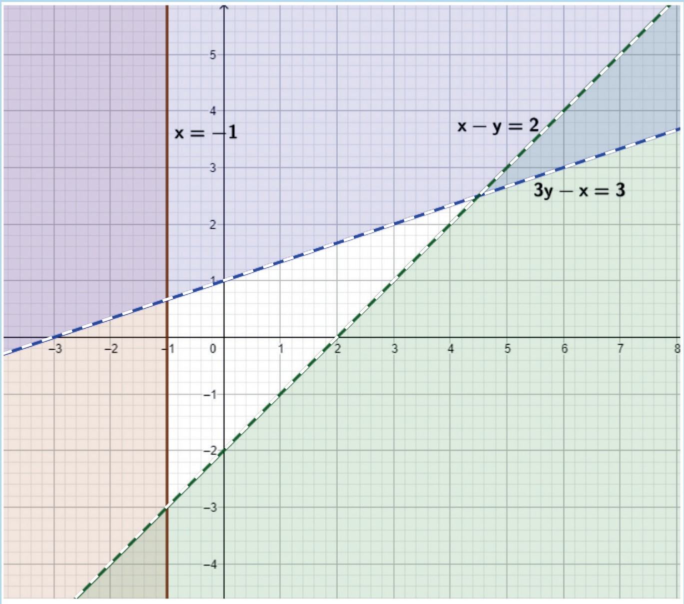
$3y - x < 3$; $x - y < 2$; $x \ge -1$.

$3y - x < 3$

x	0	–3
y	1	0

$x - y < 2$

x	0	2
y	–2	0



Quick Self-Test Questions 7.3

1. Calculate $(3x^2 + 4x + 9) + (4x^2 + 4x + 4)$
2. Subtract $8yx^2 - 4y^2x^2 + y^2x^2$ from $3yx^2 + 4y^2x^2$
3. Find the HCF and LCM of the following:
   a) 130 and 348
   b) $2x^2y^2$ and $90x^3$
4. Expand and simplify each of the following:
   a) $(x + y)(a + b)$
   b) $(3a - 4)(3a - 4)$
   c) $x^2(a + b) + (a + b)y$
5. Factorise completely:
   a) $1.15m^3 - 35m^2$
   b) $3a^2b^3 - 2b^3c^2$

Simplify

6. $4^4 \times 4^2$
7. $4^7 \div 4^3$
8. $100^{1/2}$
9. $(x^2)^2$
10. 9^{-2}
11. $\left(\frac{2}{4}\right)^{-1}$
12. Solve

    a) $8x+2=6$ b) $\frac{x}{6}+1=\frac{x}{3}$
13. Solve graphically:

    a) $4x - 3y = 1$, $2x + 4y = 17$ b) $2c - d = 11$, $c + 2d = -7$ c) $5x - 2y = 15$, $3x + 5y = 9$
14. Solve the following quadratic equations by factorisation, substitution or using the quadratic formula:

    a) $4x^2 - 49 = 0$ b) $x^2 - 12x = 2x - 45$
    c) $3x^2 - 7x - 1 = 0$ d) $7x^2 - x - 20 = 0$
15. Solve $x - 3 \leq 3x + 10$ and illustrate the solution set on a number line.
16. Solve the inequality $y - 4 < 3y + 2 \leq 6 - y$. Hence, list the integral values of y that satisfies the inequality.
17. a) Solve the simultaneous inequalities $2x - 6 < 5x + 3 \leq 3x + 11$ giving your answer in the form $a < x \leq b$ where a and b are integers.
    b) Write down the least possible value of x.

End of Topic 7 Revision Questions

Make *b* the subject of formula in each of the following:

1. $y=\dfrac{3}{b-2}$ **2.** $x=ut-\dfrac{1}{4bt^2}$ **3.** $y=\dfrac{1}{4}\sqrt{0.5b-x^2}$

4. $xy=\dfrac{1}{4}b+\dfrac{1}{2}x$ **5.** $g-\dfrac{h}{b}=0$ **6.** $\dfrac{x+b}{x+y}=0$

7. Simplify where possible;

a) $21-2 \times 5x$ b) $9a \times 3b$

c) $\frac{1}{3}\times\frac{1}{5}xy$ d) $7x \times 2 + 5 \times 8x - 6x \times 9$

e) $3x + 6x$ f) $8m + 15n$

g) $3y + 8k - 2 - 2k - y$ h) $12k + 10p - 7k$

i) $7x + 5y - 4$ j) $2x - 3y + 9 - 5k + 8y + 14$

8. Simplify the following fractions:

a) $\dfrac{1}{x-1}+\dfrac{2}{x+1}$ b) $\dfrac{6x^2ny^3}{24x^3n^2y^4}$ c) $\dfrac{x^2-9}{x^2-7x+12}$

d) $\dfrac{(2x-y)^2-(x-2y)^2}{5x^2-5y^2}$ e) $\dfrac{x+y}{x+a}$

9. Find the HCF and LCM of the following:

a) 9 and 3 b) 100 and 300 c) 56 and 28 d) $2xy$ and $90x$

e) $3xy^3z$ and $51x^2yz$ f) $66x^3yp$ and $4xz^3$ g) 13 and 39 h) 100 and 11

i) $2x$ and $90x$

10. Expand the following:

a) $(x + y)(a - b)$ b) $(x + y)(3a - 2b)$ c) $(x - xy - y)(x - y)$

d) $(3x + 2y)(2x - 4y)$ e) $(2a^2 - 2b^2)(x - y)$ f) $(a - 9)^2$

11. Factorise completely:

a) $4a^2b^2c^3 + 8ab^2c - 10a^2b^3c^3$

b) $10pqs - 15mpq - 25pqt + 30mnp$

c) $4ab - 2bc$

d) $3a^2m^2 - am$

e) $7x^2y^2z - 2x^2y^2 + 4x^2a^2y^2 - 8x^2y^2$

12. a) $3 \times 3^3 =$ b) $9^2 \times 9^4 =$ c) $(2/4)^3 \times (2/4)^4 =$ d) $(-1/4)^2 \times (-1/4)^3 =$

Simplify

13. a) $2^6 \div 2^{-2} =$ b) $\left(\frac{1}{2}\right)^6 \div \left(\frac{1}{2}\right)^4$ c) $3^8 \div 3^2$ d) $x^{\frac{3}{4}} \div x^{\frac{1}{2}}$

e) $\left(\frac{1}{4}\right)^{-1} \div \left(\frac{1}{4}\right)^{-3}$

14. a) $625^{\frac{1}{4}}$ b) $2\,744^{\frac{1}{3}}$ c) $128^{\frac{1}{7}}$ d) $81^{\frac{1}{4}}$

15. a) $(x^{10})^{\frac{1}{2}}$ b) $(x^3)^2$ c) $(x^9)^9$ d) $(k^{10})^2$

16. a) $\left(\frac{1}{2}\right)^{-1}$ b) 3^{-3} c) $\left(\frac{1}{4}\right)^{-2}$

17. a) $\left(\frac{16}{49}\right)^{\frac{1}{2}}$ b) $\left(\frac{9}{25}\right)^{-\left(\frac{1}{2}\right)}$ c) $\left(\frac{49}{81}\right)^{\frac{1}{2}}$

18. Solve the following equations:

a) $\frac{4x-5}{7}=1\frac{3}{4}$ b) $6x+5=24x+10$ c) $2x^2+3x-10=2x(x+1)$

d) $\frac{x}{2}-\frac{x}{3}=6$ e) $\frac{1}{4}+\frac{9}{x}=1$ f) $\frac{x}{2}+\frac{x}{3}=10$

g) $8x-13=x-1$ h) $\frac{x+2}{x+3}-3=\frac{1}{3-2x-x^2}$ i) $\frac{3}{y-1}+\frac{3}{y-3}=\frac{8}{y-1}$

j) $\frac{a-1}{a+3}-\frac{1-2a}{3-a}=\frac{2-a}{a-3}$ k) $\frac{11}{x}-1\frac{1}{2}=\frac{5}{x}$ l) $3\left(2x+\frac{1}{2}\right)=2x-13$

m) $\frac{2}{x+2}=\frac{1}{3}$

19. Solve the following simultaneous equations using any of the three methods:

a) $a + b = 12$
$3a - b = 6$

b) $2x + 3y = -1$
$x - \frac{1}{2}y = 1$

c) $4x - \frac{1}{4}y = 3$
$3x + y = 1$

d) $2x + 4y = \frac{1}{2}$
$3x + 5y = \frac{1}{4}$

e) $y + x = -\frac{7}{8}$
$10x = 4 - 3y$

f) $y + x = -1$
$x - y = -2$

g) $3x - 4 = y$
$6x + 8y = 1$

20. Solve giving answers correct to 2 decimal places where possible.

a) $2x^2 - 3x - 7 = 0$ b) $x^2 - 7x - 12 = 0$ c) $3x^2 + 8x - 44 = 0$

d) $2x^2 + 3x - 10 = 0$ e) $3x(x+7) = 16 - x$ f) $6y^2 + 15y = 4$

g) $x^2 + 5x + 6 = 0$ h) $x^2 = 64$ i) $x^2 + 2x - 8 = 0$

j) $\frac{7x+1}{5}=-x^2$ k) $(x-3)^2 = 4$ l) $x^2 - 3x + 2 = 0$

m) $2x^2 - 8 = 0$ n) $3x^2-4x=\frac{5}{3}$ o) $90x^2 - 90x = 30$

p) $x^2 + 12x = -36$ q) $x^2 + x - 2 = 0$ r) $x^2 + 3x + 2 = 0$

21. Solve the inequality $15-3x<2(x-5)$.

22. Solve the inequality $5x-5\leq 10x<8x+10$ and illustrate your answer on a number line.

23. If x is an integer such that $4x < 30$ and $20 - 3x \leq 5$, list the possible values of x.

Show on a graph the region represents, A, the set of points $(x; y)$ which satisfies the three inequalities $x \geq -2$; $3x + y \leq 11$; $y > -3$